Chapter 3

Mathematics Performance at the TIMSS Advanced 2008 International Benchmarks

As was described more fully in the Introduction, the TIMSS advanced mathematics achievement scale summarizes students' performance on test items designed to measure breadth of content in algebra, geometry, and calculus, as well as a range of cognitive processes within the knowing, applying, and reasoning domains. To interpret the achievement results in meaningful ways, it is important to understand the relationship between scores on the scale and students' success on the content of the assessment. As a way of interpreting the scaled results, three points on the scale were identified as international benchmarks and descriptions of student achievement at those benchmarks in relation to students' performance on the test items were developed. The TIMSS Advanced benchmarks represent the range of performance shown by students internationally. The Advanced International Benchmark is 625, the High International Benchmark is 550, and the Intermediate International Benchmark is 475. In TIMSS at the fourth and eighth grade levels, four benchmarks were used: viz., advanced, high, intermediate, and low. The low international benchmark was not included in the TIMSS Advanced benchmarking analysis since, in all the participating countries, this is a highly select population of students.

The TIMSS & PIRLS International Study Center worked with a committee of experts[1] from several countries to conduct a detailed scale anchoring analysis to describe mathematics achievement at these benchmarks. Scale anchoring is a way of describing TIMSS Advanced 2008 performance at different points on the advanced mathematics scale in terms of the types of items students answered correctly. In addition to a data analysis component to identify items that discriminated between successive points on the scale,[2] the analysis also involved a judgmental component in which committee members examined the mathematics content and cognitive processing dimensions assessed by each item and generalized to describe students' knowledge and understandings.

This chapter presents the TIMSS Advanced 2008 mathematics achievement results at the international benchmarks for the participating countries. Then, benchmark by benchmark, there is a detailed description of the understanding of mathematics content and types of cognitive processing skills and strategies demonstrated by students at each of the international benchmarks, together with illustrative items. For each example item, the percent correct for each of the TIMSS Advanced 2008 participants is shown. For multiple-choice items, the correct answer is identified by a bullet, •, and the percent of students in each country who chose each response choice is also given. For constructed-response items, a copy of the scoring guide showing the percent of students choosing each correct or incorrect approach is provided, along with a student response that was given full credit.[3] The items published in this report were selected from the items released for public use.[4] Every effort was made to include examples which not

1 In addition to Robert A. Garden, the TIMSS Advanced Mathematics Coordinator, and Svein Lie, the TIMSS Physics Coordinator, committee members included Carl Angell, Wolfgang Dietrich, Liv Sissel Gronmo, Torgeir Onstad, and David F. Robitaille.

2 For example, in brief, a multiple-choice item anchored at the Advanced International Benchmark if at least 65 percent of students scoring at 625 answered the item correctly and fewer than 50 percent of students scoring at the High International Benchmark (550) answered correctly, and so on, for each successively lower benchmark. Since constructed-response questions nearly eliminate guessing, the criterion for the constructed-response items was simply 50 percent at the particular benchmark. For more information, see the *TIMSS Advanced 2008 Technical Report*.

3 All of the constructed-response items were scored according to detailed scoring guides containing descriptions and examples of the types of responses that should receive credit. Although most constructed-response items were worth 1 point, some were worth 2 points (with 1 point awarded for partial credit). If the example item was worth 2 points, the data are for responses receiving 2 points (full credit).

4 After each TIMSS assessment, a certain proportion of the items are released into the public domain and the rest of the items are kept secure for use in measuring trends over time in subsequent assessments. In the case of TIMSS Advanced, more than one-half of the items are being released.

only illustrated the particular benchmark under discussion, but also represented different item formats and content area domains.

How Do Countries Compare on the TIMSS Advanced 2008 International Benchmarks of Mathematics Achievement?

Exhibit 3.1 summarizes what students of advanced mathematics in the participating countries who score at the TIMSS international benchmarks typically know and can do in mathematics. The data show that that there were substantial differences in students' performance across the three benchmarks. Students at the Advanced International Benchmark demonstrated their understanding of concepts, mastery of procedures, and mathematical reasoning skills in algebra, trigonometry, geometry, and differential and integral calculus to solve problems in complex contexts. Students at the High International Benchmark used their knowledge of mathematical concepts and procedures in algebra, calculus, and geometry and trigonometry to analyze and solve multi-step problems set in routine and non-routine contexts. Those at the Intermediate International Benchmark demonstrated knowledge of concepts and procedures in algebra, calculus, and geometry.

Exhibit 3.2 displays the percent of advanced mathematics students in each country that reached each of the three international benchmarks. The percents displayed in each row corresponding to the three international benchmarks are cumulative. Every student who scored at the Advanced Benchmark is also included in the High and Intermediate Benchmark categories.

For each country, the exhibit shows the percent of advanced mathematics students who reached each international benchmark as well as the TIMSS Advanced Mathematics Coverage Index for that country (see Exhibit 1.2). In the table, the countries are listed in descending order of the percent of their students who reached the

Exhibit 3.1 **TIMSS Advanced 2008 International Benchmarks of Mathematics Achievement**

TIMSSAdvanced 2008 Advanced Mathematics

Advanced International Benchmark – 625

Summary

Students demonstrate their understanding of concepts, mastery of procedures, and mathematical reasoning skills in algebra, trigonometry, geometry, and differential and integral calculus to solve problems in complex contexts.

In algebra, students can solve word problems involving permutations and geometric sequences, and solve logarithmic equations. They demonstrate some facility with complex numbers and can find sums of infinite geometric series. In calculus, students demonstrate understanding of the concept of integration. They can integrate exponential functions, recognize the relationship between a definite integral and the area under a curve, and solve problems about areas between curves. They can identify from the graph of a function points where it is not differentiable. They can determine maxima, minima, and points of inflection of a function by analyzing the graph of its derivative or by finding the first and second derivatives. They can solve problems in kinematics, and find the maximum value of a quantity under given conditions. Students use geometric reasoning to solve problems. They can use trigonometric ratios to solve a non-routine practical problem, and demonstrate knowledge of the concepts of period and amplitude of trigonometric functions. They use vector sums and differences to express a relationship among three vectors. In the Cartesian plane, they can determine whether lines are parallel, show that the diagonals of a given quadrilateral bisect each other, and find the locus of points satisfying a given condition.

High International Benchmark – 550

Summary

Students can use their knowledge of mathematical concepts and procedures in algebra, calculus, and geometry and trigonometry to analyze and solve multi-step problems set in routine and non-routine contexts.

Students can solve algebra problems that require analysis, including problems set in a practical context and problems requiring interpretation of information related to functions and their graphs. They can determine a term in a geometric sequence, compare two simple mathematical models, solve quadratic inequalities, and analyze a proposed solution of a simple logarithmic equation. In calculus, students can analyze properties of functions and their graphs on the basis of the sign of the first and second derivatives. They can find the derivative of a function involving radicals. They can find definite and indefinite integrals of simple rational functions. In geometry, students can use basic properties of trigonometric functions to identify solutions of simple trigonometric equations and solve word problems involving angle of elevation. They can identify the equation of a line or a circle in the Cartesian plane, and use slopes of lines to solve problems. They can use properties of vectors to analyze equivalence of conditions involving the sum and difference of two vectors.

Exhibit 3.1 **TIMSS Advanced 2008 International Benchmarks of Mathematics Achievement (Continued)**

TIMSSAdvanced 2008
Advanced Mathematics

Intermediate International Benchmark – 475

Summary

Students demonstrate knowledge of concepts and procedures in algebra, calculus, and geometry to solve routine problems.

Students can perform basic operations of algebra, including solving equations and inequalities, and simplifying polynomial and rational expressions. They can determine the sign of a rational function and find the function of a function in simple cases. In calculus, students show an understanding of the concepts of continuity and limit of a rational function. They can find the derivative of simple rational, exponential, and trigonometric functions. They can make connections between the graph of a function and the derivative of the function. Students use knowledge of basic properties of geometric figures and of the Cartesian plane to solve problems. They can add and subtract vectors in coordinate form. They can draw the image of a polygon under a reflection, and identify the shape traced by a line rotating in space.

Exhibit 3.2 **Percent of Students Reaching the TIMSS Advanced 2008 International Benchmarks of Mathematics Achievement**

TIMSSAdvanced2008 Advanced Mathematics

Country	Percent of Students Reaching the International Benchmarks			TIMSS Advanced Mathematics Coverage Index
	Advanced Benchmark (625)	High Benchmark (550)	Intermediate Benchmark (475)	
Russian Federation	24 (2.9)	55 (3.2)	83 (2.2)	1.4%
Iran, Islamic Rep. of	11 (1.8)	29 (3.0)	56 (2.8)	6.5%
Lebanon	9 (1.2)	47 (1.9)	88 (1.3)	5.9%
† Netherlands	6 (0.8)	52 (2.8)	95 (1.1)	3.5%
Italy	3 (1.0)	14 (2.0)	41 (3.0)	19.7%
Slovenia	3 (0.5)	14 (1.4)	41 (2.4)	40.5%
Armenia	2 (0.8)	13 (1.6)	33 (2.0)	4.3%
Norway	1 (0.4)	9 (1.0)	35 (2.2)	10.9%
Sweden	1 (0.4)	9 (1.2)	29 (1.9)	12.8%
Philippines	1 (0.3)	4 (0.7)	13 (1.5)	0.7%

† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).

( ) Standard errors appear in parentheses.

Exhibit 3.3 **Trends in Percent of Students Reaching the TIMSS Advanced 2008 International Benchmarks of Mathematics Achievement**

TIMSSAdvanced2008 Advanced Mathematics

Country	TIMSS Advanced Mathematics Coverage Index		Percent of Students Reaching the International Benchmarks					
			Advanced International Benchmark (625)		High International Benchmark (550)		Intermediate International Benchmark (475)	
	2008	1995	2008 Percent of Students	1995 Percent of Students	2008 Percent of Students	1995 Percent of Students	2008 Percent of Students	1995 Percent of Students
Russian Federation	1.4%	2.0%	24 (2.9)	22 (3.1)	55 (3.2)	51 (3.5)	83 (2.2)	78 (2.7)
Italy	19.7%	20.2%	3 (1.0)	5 (2.2)	14 (2.0)	22 (5.0)	41 (3.0) ▼	59 (4.9)
Slovenia	40.5%	75.4%	3 (0.5)	5 (1.3)	14 (1.4) ▼	23 (3.5)	41 (2.4) ▼	54 (4.5)
Sweden	12.8%	16.2%	1 (0.4) ▼	6 (1.4)	9 (1.2) ▼	30 (3.3)	29 (1.9) ▼	64 (3.1)

▲ 2008 percent significantly higher than 1995
▼ 2008 percent significantly lower than 1995

( ) Standard errors appear in parentheses.

Advanced Benchmark. As might be expected, given that it had the highest mathematics achievement average, the Russian Federation had the greatest percentage of students (24%) reaching the Advanced International Benchmark. Next came Iran with 11 percent, then Lebanon with 9 percent, and the Netherlands with 6 percent. It is noteworthy that relatively more students reached the Advanced Benchmark in Iran and the Lebanon than in the Netherlands, even though average achievement was higher in the Netherlands. This is a reflection of the relatively narrow range of achievement in the Netherlands, evident in Exhibit 2.1, compared to most other participating countries. A more positive consequence of the Netherlands narrow achievement range is that it had the highest percentage of students (95%) reaching the Intermediate Benchmark.

The percent of students who scored at the Intermediate Benchmark ranges from a low of 13 percent in the Philippines to a high of 95 percent in the Netherlands. Results for Slovenia and Italy indicate that countries with a comparatively high TIMSS Advanced Mathematics Coverage Index are still able to obtain strong performance from many of their students. These results show that a system-wide policy of allowing a larger proportion of students to enroll in advanced courses in mathematics does not necessarily have a negative impact on overall students' performance. It can provide opportunities for further study in mathematics-related specialty areas to more students. In all of these kinds of comparisons, it is important to bear in mind the potential impact of the Mathematics Coverage Index on performance levels.

On the one hand, these students—the very best mathematics students in their respective countries—found the TIMSS advanced mathematics test to be challenging. In six countries the percent of students reaching the Advanced Benchmark was 3 percent or less. On the other hand, in six countries, more than 40 percent of students reached at least the Intermediate Benchmark which, as shown in Exhibit 3.1, means that those students demonstrated knowledge of the concepts and procedures in algebra, calculus, and geometry assessed by TIMSS Advanced 2008.

Exhibit 3.3 presents changes in the percent of students reaching the benchmarks between 1995 and 2008 for the four countries that participated in both studies. Countries are ranked in descending order of the percent of students who reached the Advanced International Benchmark. The display also shows the TIMSS Advanced Mathematics Coverage Index for each country in the 1995 and 2008 assessments. Over that period, the index declined in all four countries. The most dramatic drop in the Coverage Index occurred in Slovenia: from 75 percent coverage in 1995 to 40 percent in 2008.

The results reflect the overall changes in achievement for the four countries, with all experiencing declines since 1995 except the Russian Federation, which evidenced little, if any, change (see Exhibit 2.4). No country showed a significant improvement in the percent of students reaching any of the three international benchmarks. However, there were several significant declines. Sweden experienced declines at all three benchmarks even though the population appears to have become more specialized between 1995 and 2008. Also, Slovenia, with the broadest population coverage but still greatly reduced in scope compared to 1995, had significantly fewer students reaching the High and Intermediate Benchmarks. Italy had declines at the Intermediate Benchmark.

Mathematics: Achievement at the Advanced International Benchmark

The *TIMSS Advanced 2008 Assessment Frameworks* called for an almost equal partitioning of the items to be included in the advanced mathematics assessment among the three content domains: 35 percent for algebra, 35 percent for calculus, and 30 percent for geometry.

According to the framework, the algebra content domain includes much of the algebra and functions content that provides the foundation for mathematics at the college or university level. Students should be able to use properties of the real and complex number systems to solve problems set in real-world contexts or in abstract, mathematical ones. They should have facility in investigating basic characteristics of sequences and series, and skill in manipulating and using combinations and permutations. The ability to work with a variety of equations is fundamental for such students, providing a means of operating with mathematical concepts at an abstract level. The concept of function is an important unifying idea in mathematics, and students should be familiar with it.

Since the calculus content of national and system-level advanced mathematics curricula varies considerably across countries, the calculus content for TIMSS Advanced Mathematics 2008 was limited to material likely to be included in final year mathematics in almost all the participating countries. The focus was on understanding limits and finding the limit of a function, differentiation and integration of a range of functions, and using these skills in solving problems.

The TIMSS geometry items related to four strands or topics: Euclidean geometry (traditional or transformation), analytic geometry, trigonometry, and vectors. Euclidean geometry and analytic geometry have been important components of the secondary mathematics curriculum for centuries, and are still widely viewed as important prerequisites for the study of mathematics at the university level. Trigonometry is part of the mathematics curriculum in all countries, but not always as part of the geometry domain. Transformation geometry and vectors are more recent additions to the mathematics curriculum in many countries, and there is considerable variation both in the amount of emphasis given to them across countries, as well as the degree of rigor with which the area is approached. The TIMSS items related to these two areas dealt with fairly elementary topics.

In the algebra domain, the framework specifies that students should recognize representations of functions and be able to solve various kinds of equations, including quadratic equations. Exhibit 3.4 presents an algebra item likely to be solved correctly by students performing at the Advanced International Benchmark. In this example (Example Item 1), students were asked to find the numerical coefficients of a quadratic function having been given its graph and its x- and y-intercepts. An example of a correct solution to this constructed-response item is shown in the exhibit. According to the information provided in Chapter 1 on the topics that were in the intended curriculum and taught to the students (Exhibit 1.13), all countries included polynomial equations and functions in their curriculum, and taught these topics (except function of a function in the Philippines) to their students. Nevertheless, students found this item difficult, and this was true for all of the items that anchored at the Advanced Benchmark. The percent of students receiving full credit ranged from a high of 64 in Lebanon to a low of 8 in Sweden. After Lebanon, the next highest result was 39 percent correct in the Russian Federation.

The scoring guide for Example Item 1 shows the five correct- and the four incorrect-response categories used by the item scorers as well as the non-response category. Also shown are the percents of students in each category in each country. Category 13 refers to the use of a graphing calculator to find the coefficients of the equation. The total percent correct for a given country is the sum across the various correct-response categories.

The most frequently used correct solution method for Example 1, in every country except Armenia, was using simultaneous linear equations in three variables (*a, b,* and *c*) given three pairs of values for *x* and *f(x)*. The other four correct approaches were used by very few students. Non-response rates for this item ranged from a low of 10 percent in Lebanon, the country with the highest score on the item, to 55 percent in Sweden, 63 percent in Norway, and 70 percent in Armenia. The category 72 indicates that many students in some countries were able to find the value of the constant term, *c*, but not of *a* or *b*.

Exhibit 3.5 shows an example multiple-choice item from the calculus domain that anchored at the Advanced Benchmark (Example Item 2). The item was designed to test students' understanding of the definite integral, and the alternatives were chosen to reflect common errors or misconceptions. Students had to realize that the definite integral was not simply the sum of the three shaded areas, but the "signed" or algebraic sum, where the value of area B was negative. Not surprisingly, the incorrect response most frequently chosen in most countries was 7.6, the sum of the absolute values of the three areas identified on the graph.

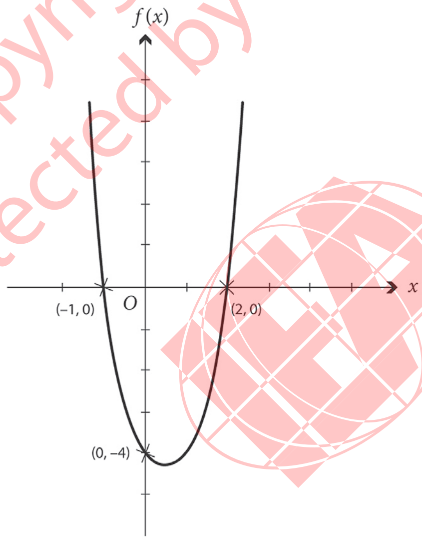
The percent correct in every country was rather low, and there was not as much variation in the proportion of students selecting the correct response across countries as was the case with many other items. The highest performance on this item was 46 percent in the Islamic Republic of Iran and 41 percent correct in the Russian Federation. About one-third of the students responded correctly in the Netherlands, Lebanon, and Slovenia. Understandably, the lowest

Exhibit 3.4 **TIMSS Advanced 2008 Advanced International Benchmark (625) of Mathematics Achievement – Example Item 1**

TIMSSAdvanced2008 Advanced Mathematics

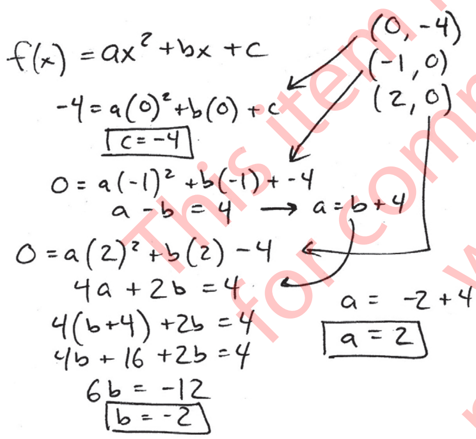
Content Domain: Algebra

Description: Determines the coefficients of a quadratic function given the points of intersection between the graph and the axes



The graph of the function *f* is shown above. The equation of the function *f* is given by $f(x) = ax^2 + bx + c$. Find the values of *a*, *b*, and *c*.

Show your work.



The answer shown is an example of a student response that was scored as correct

Country	Percent Correct
Lebanon	64 (2.9)
Russian Federation	39 (2.7)
Slovenia	32 (2.5)
Iran, Islamic Rep. of	32 (2.7)
Italy	22 (2.8)
† Netherlands	16 (1.8)
Armenia	16 (2.7)
Norway	10 (1.6)
Philippines	9 (1.7)
Sweden	8 (1.8)

SOURCE: IEA TIMSS Advanced 2008 ©

† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).

( ) Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Exhibit 3.4 **TIMSS Advanced 2008 Advanced International Benchmark (625) of Mathematics Achievement – Example Item 1 (Continued)**

TIMSSAdvanced2008 Advanced Mathematics

Scoring Guide

Code	Response	Item: MA23141
	Correct Student Responses	
10	$a = 2$, $b = -2$, $c = -4$ using factorization	
11	All values correct by solving three simultaneous equations	
12	All values correct using calculator to solve simultaneous equations	
13	All values correct using calculator for quadratic regression	
19	All values correct by other correct method.	
	Incorrect Student Responses	
70	Calculator used but incorrect or explanation inadequate	
71	All values correct but no correct method shown.	
72	$c = -4$ with values of a and b missing or incorrect.	
79	Other incorrect	
NR	No Response	

Country	Percent of Students in Each Scoring Guide Category									
	Correct Student Responses					Incorrect Student Responses				
	10	11	12	13	19	70	71	72	79	NR
Lebanon	8 (1.7)	54 (2.9)	2 (0.7)	0 (0.2)	1 (0.4)	1 (0.6)	17 (2.1)	0 (0.0)	7 (1.6)	10 (1.7)
Russian Federation	1 (0.4)	31 (2.7)	0 (0.0)	0 (0.0)	6 (1.2)	0 (0.0)	2 (0.8)	14 (1.8)	14 (1.2)	31 (2.7)
Slovenia	8 (2.0)	24 (2.3)	0 (0.0)	0 (0.0)	1 (0.4)	0 (0.0)	0 (0.3)	28 (2.4)	23 (2.1)	16 (2.1)
Iran, Islamic Rep. of	1 (0.4)	29 (2.6)	0 (0.0)	0 (0.0)	1 (0.5)	0 (0.0)	1 (0.5)	12 (1.6)	15 (1.9)	40 (2.9)
Italy	7 (1.6)	14 (2.4)	0 (0.1)	0 (0.0)	0 (0.0)	0 (0.3)	1 (0.4)	12 (2.1)	7 (1.1)	58 (3.5)
† Netherlands	1 (0.6)	11 (1.8)	0 (0.0)	1 (0.7)	2 (0.7)	1 (0.6)	2 (0.7)	30 (2.2)	27 (2.6)	23 (2.1)
Armenia	8 (2.4)	6 (2.1)	0 (0.0)	0 (0.0)	1 (0.7)	0 (0.0)	0 (0.0)	5 (2.0)	10 (3.0)	70 (3.2)
Norway	1 (0.4)	1 (0.5)	0 (0.1)	6 (1.5)	1 (0.5)	2 (0.7)	1 (0.4)	9 (1.5)	15 (1.4)	63 (3.1)
Philippines	2 (0.5)	7 (1.3)	0 (0.1)	0 (0.0)	0 (0.0)	0 (0.2)	0 (0.1)	8 (1.0)	49 (2.3)	34 (2.5)
Sweden	2 (0.6)	5 (1.4)	0 (0.4)	0 (0.3)	0 (0.3)	0 (0.3)	0 (0.0)	18 (2.0)	19 (1.8)	55 (2.5)

† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).

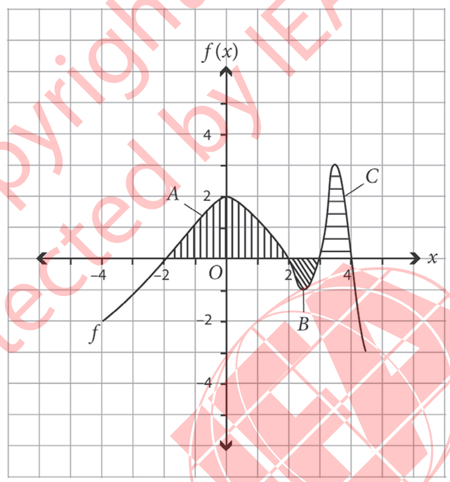
( ) Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Exhibit 3.5 **TIMSS Advanced 2008 Advanced International Benchmark (625) of Mathematics Achievement – Example Item 2**

TIMSSAdvanced 2008 Advanced Mathematics

Content Domain: Calculus

Description: Calculates the definite integral given the graph of a function and the areas between the curve and the x-axis



For the areas between the graph of $f(x)$ and the x-axis shown above, area $A = 4.8$ units, area $B = 0.8$ units, and area $C = 2$ units.

What is the value of the definite integral $\int_{-2}^{4} f(x)dx$?

Ⓐ 5.6

● 6.0

Ⓒ 6.8

Ⓓ 7.6

Country	Percent Correct
Iran, Islamic Rep. of	46 (3.1)
Russian Federation	41 (3.3)
† Netherlands	36 (2.6)
Lebanon	35 (2.7)
Slovenia	32 (2.7)
Italy	26 (2.8)
Sweden	26 (1.7)
Norway	23 (1.9)
Philippines	23 (1.8)
Armenia	18 (3.2)

Country	Percent of Students				
	A	B Correct Response	C	D	NR*
Iran, Islamic Rep. of	3 (0.5)	46 (3.1)	6 (1.1)	12 (1.6)	32 (2.5)
Russian Federation	5 (0.8)	41 (3.3)	14 (1.4)	29 (2.2)	11 (1.3)
† Netherlands	4 (1.1)	36 (2.6)	13 (1.5)	30 (2.8)	18 (2.3)
Lebanon	3 (0.6)	35 (2.7)	7 (1.3)	36 (2.1)	19 (2.0)
Slovenia	3 (0.7)	32 (2.7)	15 (1.7)	28 (3.5)	21 (2.1)
Italy	5 (1.2)	26 (2.8)	14 (2.0)	20 (2.3)	34 (3.2)
Sweden	11 (1.1)	26 (1.7)	21 (1.8)	20 (1.9)	21 (2.1)
Norway	4 (1.1)	23 (1.9)	19 (2.4)	36 (2.3)	18 (1.6)
Philippines	12 (1.6)	23 (1.8)	24 (1.5)	35 (1.8)	6 (0.9)
Armenia	7 (1.9)	18 (3.2)	14 (2.9)	9 (2.3)	53 (3.7)

* No Response

† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).

( ) Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

performance (18%) was in Armenia where this topic is not included in the advanced curriculum. Non-response rates for this item ranged from a low of 6 percent in the Philippines to a high of 53 percent in Armenia.

The third example of an item that anchored at the Advanced Benchmark comes from the geometry domain and is shown in Exhibit 3.6. Example Item 3 required students to solve a multi-step word problem involving trigonometric ratios to identify the length of a side of a regular polygon inscribed in a circle. All participants included trigonometry in their intended curriculum, and teachers reported teaching these topics to nearly all students in their advanced mathematics classes (94–100%). The problem was posed in a situation that was practical, yet novel for most students.

The best performance on this item was in the Russian Federation where 40 percent of students selected the correct response. In 6 of the 10 countries, the average percent correct was at the chance level, 25 percent, or lower. One method of solving this problem would be to drop a perpendicular bisector from the center of the circle to the base of the triangle formed by a pair of adjacent radii and one of the windows. The perpendicular divides the triangle into two right triangles, and the length of the base of each of those triangles is $r \sin 9°$. A second method would involve the use of the sine law.

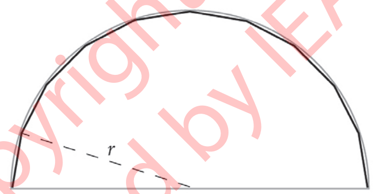
Non-response rates for this item were quite low in most countries, and response C was the most common incorrect response in all countries except the Islamic Republic of Iran. All three alternatives attracted significant numbers of students in all countries.

Exhibit 3.6 **TIMSS Advanced 2008 Advanced International Benchmark (625) of Mathematics Achievement – Example Item 3**

TIMSSAdvanced 2008 Advanced Mathematics

Content Domain: Geometry

Description: Solves a multi-step word problem involving trignometric ratios to identify the length of a side of a regular polygon inscribed in a circle



The figure shows a semicircular room seen from above. An architect is placing 10 flat windows in the room as shown. If the radius of the circle is r, which of the following equations would allow the architect to determine the width of each window?

Ⓐ $w = r \sin 9°$

● $w = 2r \sin 9°$

Ⓒ $w = r \cos 18°$

Ⓓ $w = 2r \sin 18°$

Country	Percent Correct
Russian Federation	40 (2.4)
† Netherlands	36 (2.7)
Iran, Islamic Rep. of	28 (2.4)
Slovenia	26 (2.0)
Lebanon	25 (2.5)
Italy	22 (2.5)
Sweden	22 (1.7)
Philippines	21 (1.4)
Armenia	20 (3.1)
Norway	18 (1.8)

Country	Percent of Students				
	A	B Correct Response	C	D	NR*
Russian Federation	10 (1.3)	40 (2.4)	25 (1.8)	22 (1.7)	3 (0.6)
† Netherlands	8 (1.4)	36 (2.7)	32 (2.4)	22 (2.2)	2 (0.8)
Iran, Islamic Rep. of	11 (1.5)	28 (2.4)	15 (1.9)	22 (2.1)	24 (1.9)
Slovenia	10 (1.1)	26 (2.0)	40 (2.1)	20 (2.0)	4 (1.1)
Lebanon	11 (1.6)	25 (2.5)	29 (2.4)	22 (2.6)	13 (1.8)
Italy	12 (2.0)	22 (2.5)	28 (2.5)	21 (1.8)	16 (3.0)
Sweden	10 (1.4)	22 (1.7)	42 (2.2)	22 (1.7)	4 (0.8)
Philippines	21 (1.5)	21 (1.4)	36 (1.5)	21 (1.4)	1 (0.3)
Armenia	9 (2.8)	20 (3.1)	26 (3.3)	18 (2.4)	27 (2.8)
Norway	13 (1.5)	18 (1.8)	42 (1.9)	22 (1.9)	4 (1.0)

* No Response

† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).

( ) Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Mathematics: Achievement at the High International Benchmark

Exhibit 3.7 shows a multiple-choice item from the algebra domain that anchored at the High International Benchmark. Example Item 4 required students to identify which of four given graphs represented the relationship between the volume of a sphere and its diameter. Performance on this item was best in the Netherlands, where 60 percent of students recognized that the correct response was the only one showing that the volume of a sphere increases monotonically without an upper bound in a non-linear fashion as its diameter increases. In more than half of the countries, the percent of students responding correctly was below 40 percent. The three alternatives all attracted significant numbers of students, and the non-response rates were quite low: 7 percent or less in 9 countries and 13 percent in Armenia.

Example Item 5, shown in Exhibit 3.8, is from the calculus domain and also anchored at the High International Benchmark. This constructed-response item showed students the graph of a trigonometric function and asked why the slopes of the tangent to the graph at two given points were equal. In order to answer the item correctly, students had to know that the slope of the tangent to a curve is given by the first derivative of the function. Then they had to calculate the derivative of the given function, , and know the values of $\sin \pi$ and $\sin 2\pi$. It is not possible to tell from the incorrect response categories for this item what specific kinds of errors students made most frequently.

Students from the Netherlands had the best result on this item (53% correct, and only 3% non-response), but there was a considerable range across countries and the percent correct in six countries was less than 25. Referencing Exhibit 1.14 from Chapter 1, it can be seen that although all participants included derivatives in the intended

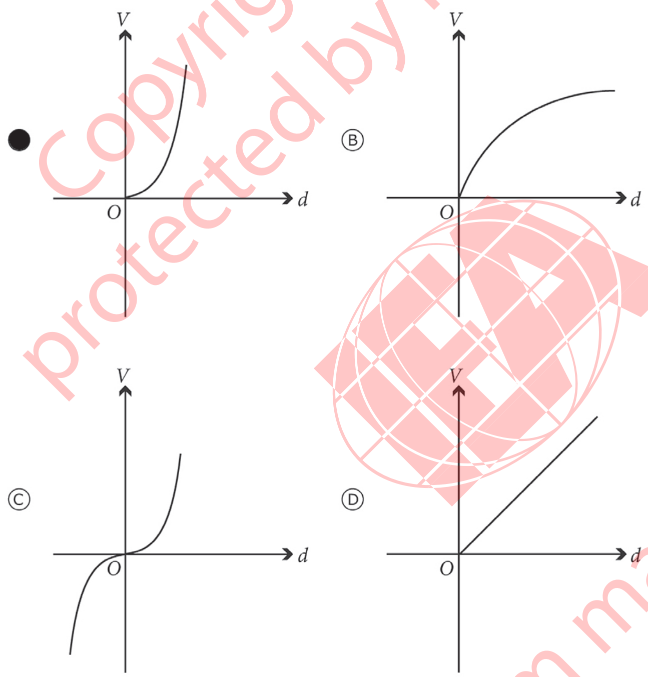
Exhibit 3.7 **TIMSS Advanced 2008 High International Benchmark (550) of Mathematics Achievement – Example Item 4**

TIMSSAdvanced2008 Advanced Mathematics

Content Domain: Algebra

Description: Identifies the graph that represents the relationship between the volume of a sphere and its diameter

A spherical balloon is blown up. Which graph shows the volume V as a function of the diameter d?



Country	Percent Correct
† Netherlands	60 (2.8)
Russian Federation	49 (2.7)
Iran, Islamic Rep. of	47 (2.9)
Sweden	42 (2.9)
Italy	38 (2.9)
Norway	37 (2.3)
Philippines	34 (2.0)
Armenia	31 (3.6)
Lebanon	30 (2.2)
Slovenia	29 (2.3)

Country	Percent of Students				
	A Correct Response	B	C	D	NR*
† Netherlands	60 (2.8)	21 (1.8)	10 (1.6)	9 (1.5)	0 (0.0)
Russian Federation	49 (2.7)	9 (1.6)	15 (2.4)	25 (1.8)	1 (0.4)
Iran, Islamic Rep. of	47 (2.9)	10 (1.6)	19 (2.0)	17 (2.1)	7 (1.3)
Sweden	42 (2.9)	27 (2.7)	9 (1.2)	21 (1.7)	2 (0.6)
Italy	38 (2.9)	17 (2.0)	10 (2.1)	30 (2.3)	5 (1.2)
Norway	37 (2.3)	23 (2.0)	16 (1.9)	23 (1.7)	1 (0.4)
Philippines	34 (2.0)	21 (1.4)	11 (1.2)	33 (1.8)	1 (0.3)
Armenia	31 (3.6)	29 (3.6)	14 (2.9)	13 (2.3)	13 (1.7)
Lebanon	30 (2.2)	31 (2.5)	13 (1.9)	19 (2.1)	7 (1.3)
Slovenia	29 (2.3)	29 (2.3)	7 (1.6)	34 (2.0)	1 (0.5)

* No Response

† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).

( ) Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

curriculum, this topic was not always covered in the implemented curriculum, with about 81 percent of the students in Lebanon taught the topic, about two-thirds in Armenia and Slovenia, and about half in the Philippines. Non-response rates varied widely across countries, and in Italy and Armenia more than 60 percent of students failed to provide an answer to this item.

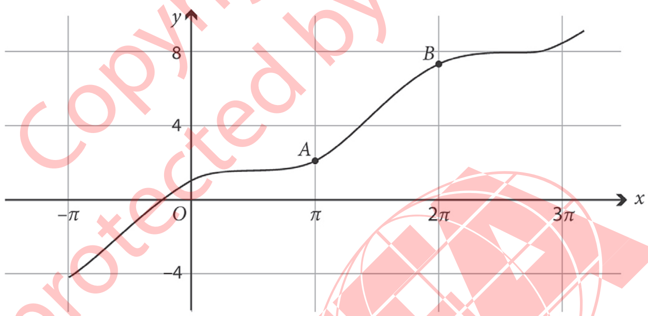
The third example of an item that anchored at the High Benchmark, Example Item 6, is from the geometry domain and is shown in Exhibit 3.9. To solve this multiple-choice item, students had to be familiar with some basic properties of the slopes of lines. Again, students from the Netherlands had the best performance on this item with 75 percent responding correctly. For 6 of the 10 countries, the percentage responding correctly was above 50 percent. Responses C and D were the most frequently chosen incorrect responses.

Exhibit 3.8 **TIMSS Advanced 2008 High International Benchmark (550) of Mathematics Achievement – Example Item 5**

TIMSSAdvanced2008 Advanced Mathematics

Content Domain: Calculus

Description: Justifies a statement about slopes at two points on the graph of a trigonometric function



Sophia is studying the graph of the function $y = x + \cos x$ shown above. She says that the slope at point A is the same as the slope at point B. Explain why she is correct.

SLOPE = $\frac{dy}{dx}$ = 1 - SIN x

= 1 - SIN π

= 1 - 0 = 1

AT 2π = 1 - SIN x

= 1 - SIN 2π

= 1 - 0 = 1

SOPHIA IS CORRECT AS SLOPE OF THIS FUNCTION IS 1 AT BOTH POINT A AND POINT B.

The answer shown is an example of a student response that was scored as correct

Country	Percent Correct
† Netherlands	53 (2.7)
Lebanon	48 (2.7)
Iran, Islamic Rep. of	45 (2.8)
Russian Federation	39 (3.3)
Sweden	22 (2.5)
Italy	19 (2.7)
Armenia	18 (2.7)
Slovenia	10 (1.5)
Norway	9 (1.2)
Philippines	2 (1.0)

SOURCE: IEA TIMSS Advanced 2008 ©

† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).

( ) Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Exhibit 3.8 **TIMSS Advanced 2008 High International Benchmark (550) of Mathematics Achievement – Example Item 5 (Continued)**

TIMSSAdvanced2008 Advanced Mathematics

Scoring Guide

Code	Response	Item: MA23198
	Correct Student Responses	
10	Differentiates or uses the cosine function to show gradient the same at $x = \pi$ and $x = 2\pi$	
11	Correct answer using calculator	
	Incorrect Student Responses	
70	Calculator used—answer incorrect or explanation inadequate	
71	Differentiates correctly—explanation inadequate	
79	Other incorrect	
NR	No Response	

Country	Percent of Students in Each Scoring Guide Category					
	Correct Student Responses		Incorrect Student Responses			
	10	11	70	71	79	NR
† Netherlands	52 (2.9)	0 (0.5)	0 (0.0)	3 (0.9)	41 (2.8)	3 (0.8)
Lebanon	48 (2.7)	0 (0.0)	3 (0.7)	0 (0.0)	33 (2.4)	16 (2.4)
Iran, Islamic Rep. of	45 (2.8)	0 (0.0)	1 (0.6)	1 (0.4)	38 (2.6)	15 (1.7)
Russian Federation	39 (3.3)	0 (0.0)	0 (0.0)	2 (0.5)	37 (2.1)	22 (2.3)
Sweden	21 (2.5)	0 (0.1)	0 (0.4)	4 (0.6)	56 (2.3)	19 (1.9)
Italy	18 (2.8)	1 (0.0)	0 (0.0)	2 (0.8)	11 (1.5)	69 (3.2)
Armenia	18 (2.7)	0 (0.0)	0 (0.0)	1 (0.0)	20 (3.0)	61 (3.9)
Slovenia	10 (1.5)	0 (0.0)	0 (0.0)	2 (0.5)	64 (2.4)	24 (2.5)
Norway	9 (1.2)	0 (0.0)	0 (0.0)	1 (0.4)	61 (2.2)	30 (2.5)
Philippines	2 (1.0)	0 (0.0)	0 (0.0)	0 (0.0)	71 (1.8)	27 (1.6)

† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).

( ) Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Exhibit 3.9 **TIMSS Advanced 2008 High International Benchmark (550) of Mathematics Achievement – Example Item 6**

TIMSSAdvanced 2008 Advanced Mathematics

Content Domain: Geometry

Description: Finds the sum of the slopes of the three sides of an equilateral triangle with one side along the x-axis

One side of an equilateral triangle lies along the x-axis. The sum of the slopes of the three sides is

- ● 0
- Ⓑ −1
- Ⓒ 1
- Ⓓ $2\sqrt{3}$
- Ⓔ $1+2\sqrt{3}$

Country	Percent Correct
† Netherlands	75 (1.5)
Iran, Islamic Rep. of	61 (2.3)
Lebanon	54 (2.0)
Slovenia	53 (2.0)
Russian Federation	52 (2.5)
Norway	51 (2.1)
Sweden	45 (1.8)
Italy	42 (2.3)
Armenia	33 (2.2)
Philippines	29 (1.7)

Country	Percent of Students				
	A Correct Response	B	C	D	NR*
† Netherlands	75 (1.5)	1 (0.5)	6 (0.9)	10 (1.1)	4 (0.6)
Iran, Islamic Rep. of	61 (2.3)	2 (0.5)	6 (0.9)	9 (1.0)	4 (0.7)
Lebanon	54 (2.0)	3 (0.5)	9 (1.1)	17 (1.5)	7 (0.9)
Slovenia	53 (2.0)	3 (0.6)	18 (1.4)	14 (1.6)	5 (0.7)
Russian Federation	52 (2.5)	3 (0.6)	11 (1.0)	22 (1.5)	6 (0.9)
Norway	51 (2.1)	1 (0.4)	18 (1.7)	14 (1.1)	6 (0.9)
Sweden	45 (1.8)	6 (0.8)	21 (1.5)	13 (1.3)	7 (0.7)
Italy	42 (2.3)	3 (0.5)	10 (1.0)	15 (1.5)	6 (0.7)
Armenia	33 (2.2)	6 (1.2)	11 (1.5)	19 (2.6)	9 (1.9)
Philippines	29 (1.7)	4 (0.4)	21 (1.2)	26 (1.2)	19 (1.3)

* No Response

† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).

( ) Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Mathematics: Achievement at the Intermediate International Benchmark

Example Item 7, shown in Exhibit 3.10, is taken from the algebra domain. This constructed-response item required students to solve an inequality involving a rational expression in one variable. All countries included inequalities in their curricula, and teachers reported that nearly all students had been taught this topic (96–100%). In the Russian Federation, 80 percent of students responded correctly. In half the countries, the percent of students providing correct responses was greater than 50. Students were not required to show their work, and it is not possible to tell from the scoring guide how students attempted to solve the inequality.

The calculus item shown in Exhibit 3.11 is a constructed-response item requiring students to find the derivative of a rational function (Example Item 8). To find this derivative, students had to know and be able to apply the quotient rule. Students in several countries performed very well on this item, with the best performance being registered in Lebanon with 91 percent of students obtaining full credit for the item. Approximately three fourths of the Iranian and Russian students as well as two thirds of the Slovenian students also received full credit. On the other hand, students in Norway, the Philippines, and Sweden found the item much more difficult. The most frequent incorrect response in several countries was based on an attempt to use the quotient rule for differentation, but doing so incorrectly.

Example Item 9, a multiple-choice item shown in Exhibit 3.12, is taken from the geometry domain. One way to solve this problem is to visualize or draw a right triangle, and recall that the vertices of a right triangle can be inscribed in a circle with the hypotenuse, being the diameter of the circumcircle. This means that *T*, the mid-point of

Exhibit 3.10 **TIMSS Advanced 2008 Intermediate International Benchmark (475) of Mathematics Achievement – Example Item 7**

TIMSSAdvanced2008 Advanced Mathematics

Content Domain: Algebra

Description: Solves a rational inequality with linear numerator and denominator

$$\frac{x+1}{x-2} > 1$$

For which values of x is the inequality shown above satisfied?

Answer: $x > 2$

The answer shown is an example of a student response that was scored as correct

Country	Percent Correct
Russian Federation	80 (1.8)
Armenia	74 (2.6)
Italy	60 (3.7)
Iran, Islamic Rep. of	54 (2.5)
Lebanon	51 (2.4)
† Netherlands	47 (2.4)
Sweden	30 (2.4)
Slovenia	26 (2.6)
Norway	16 (1.7)
Philippines	15 (1.7)

† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).

( ) Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Exhibit 3.10 **TIMSS Advanced 2008 Intermediate International Benchmark (475) of Mathematics Achievement – Example Item 7 (Continued)**

TIMSSAdvanced 2008 Advanced Mathematics

Scoring Guide

Code	Response	Item: MA23135
	Correct Student Response	
10	$x > 2$	
	Incorrect Student Responses	
79	Incorrect	
NR	No Response	

Country	Percent of Students in Each Scoring Guide Category		
	Correct Student Response	Incorrect Student Responses	
	10	79	NR
Russian Federation	80 (1.8)	19 (1.7)	1 (0.4)
Armenia	74 (2.6)	21 (2.1)	4 (1.3)
Italy	60 (3.7)	34 (3.3)	7 (1.4)
Iran, Islamic Rep. of	54 (2.5)	42 (2.5)	4 (0.9)
Lebanon	51 (2.4)	46 (2.3)	3 (1.0)
† Netherlands	47 (2.4)	48 (2.5)	5 (1.2)
Sweden	30 (2.4)	60 (2.2)	10 (1.4)
Slovenia	26 (2.6)	71 (2.7)	3 (1.1)
Norway	16 (1.7)	64 (2.1)	20 (2.0)
Philippines	15 (1.7)	78 (1.6)	8 (0.9)

† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).

( ) Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Exhibit 3.11 **TIMSS Advanced 2008 Intermediate International Benchmark (475) of Mathematics Achievement – Example Item 8**

TIMSSAdvanced2008 Advanced Mathematics

Content Domain: Calculus

Description: Differentiates a rational function where the numerator and denominator are both linear

Find $f'(x)$, when $f(x)=\frac{3x+2}{x-1}$.

Show your work.

$$f'(x) = \frac{(x-1)\frac{d}{dx}(3x+2) - (3x+2)\frac{d}{dx}(x-1)}{(x-1)^2}$$

$$= \frac{3(x-1) - (3x+2)(1)}{(x-1)^2}$$

$$= \frac{3x - 3 - 3x - 2}{(x-1)^2}$$

$$f'(x) = \frac{-5}{(x-1)^2}$$

The answer shown is an example of a student response that was scored as correct

Country	Percent Correct
Lebanon	91 (1.6)
Iran, Islamic Rep. of	79 (2.2)
Russian Federation	75 (2.4)
Slovenia	67 (2.1)
Italy	60 (3.4)
Armenia	56 (3.6)
† Netherlands	48 (2.9)
Norway	29 (2.1)
Philippines	21 (2.1)
Sweden	20 (1.8)

† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).

( ) Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Exhibit 3.11 **TIMSS Advanced 2008 Intermediate International Benchmark (475) of Mathematics Achievement – Example Item 8 (Continued)**

TIMSSAdvanced 2008 Advanced Mathematics

Scoring Guide

Code	Response	Item: MA23159
	Correct Student Responses	
10	$f'(x) = \frac{-5}{(x-1)^2}$ using $\left(\frac{u}{v}\right)' = \frac{(u'v - uv')}{v^2}$ or, $(uv)' = u'v + uv'$	
11	Correct expression using calculator	
	Incorrect Student Responses	
70	Calculator used—answer incorrect or explanation inadequate	
71	Correct answer—no working shown	
72	Using quotient rule but no correct expression	
73	Using product rule but no correct expression	
79	Other incorrect	
NR	No Response	

Note: Students were instructed that if they used a calculator they were to explain how and for what it was used.

Country	Percent of Students in Each Scoring Guide Category							
	Correct Student Responses		Incorrect Student Responses					
	10	11	70	71	72	73	79	NR
Lebanon	91 (1.6)	0 (0.0)	0 (0.2)	4 (1.1)	0 (0.0)	0 (0.0)	4 (1.0)	1 (0.6)
Iran, Islamic Rep. of	79 (2.2)	0 (0.0)	0 (0.0)	0 (0.1)	10 (1.2)	0 (0.0)	9 (1.7)	2 (0.9)
Russian Federation	75 (2.4)	0 (0.0)	0 (0.0)	0 (0.0)	8 (1.7)	0 (0.0)	14 (2.1)	3 (0.6)
Slovenia	67 (2.1)	0 (0.0)	0 (0.0)	0 (0.0)	10 (1.3)	0 (0.0)	21 (1.7)	3 (0.8)
Italy	60 (3.4)	0 (0.1)	0 (0.0)	0 (0.0)	11 (1.6)	0 (0.0)	17 (2.7)	13 (2.1)
Armenia	55 (3.6)	2 (1.0)	1 (0.6)	0 (0.0)	2 (1.1)	0 (0.0)	25 (3.3)	15 (2.0)
† Netherlands	48 (2.9)	0 (0.0)	0 (0.0)	0 (0.0)	40 (2.9)	4 (1.1)	7 (1.3)	1 (0.4)
Norway	29 (2.2)	0 (0.3)	0 (0.2)	0 (0.0)	33 (3.0)	0 (0.0)	30 (2.9)	8 (1.4)
Philippines	21 (2.1)	0 (0.0)	0 (0.0)	0 (0.0)	10 (1.2)	0 (0.0)	57 (2.3)	12 (1.6)
Sweden	19 (1.7)	0 (0.2)	0 (0.0)	0 (0.0)	19 (2.0)	3 (0.9)	48 (2.3)	10 (1.4)

† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).

( ) Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

Exhibit 3.12 TIMSS Advanced 2008 Intermediate International Benchmark (475) of Mathematics Achievement – Example Item 9

TIMSSAdvanced 2008 Advanced Mathematics

Content Domain: Geometry

Description: Uses properties of an isosceles right triangle to determine the length of a given median

Triangle *PQR* is an isosceles right triangle with a right angle at *P*. If *PT* is a median of the triangle, then *PT* has the same length as

Ⓐ *PR*

Ⓑ *PQ*

Ⓒ *QR*

● *QT*

Country	Percent Correct
Lebanon	90 (1.4)
Russian Federation	87 (1.3)
† Netherlands	79 (1.7)
Iran, Islamic Rep. of	74 (1.8)
Italy	65 (2.2)
Slovenia	63 (2.0)
Armenia	60 (2.5)
Norway	49 (1.8)
Philippines	47 (1.8)
Sweden	41 (1.2)

Country	Percent of Students				
	A	B	C	D Correct Response	NR*
Lebanon	2 (0.5)	3 (0.7)	3 (0.7)	90 (1.4)	2 (0.5)
Russian Federation	4 (0.7)	3 (0.5)	5 (0.7)	87 (1.3)	1 (0.3)
† Netherlands	4 (0.8)	4 (0.8)	10 (1.2)	79 (1.7)	4 (0.7)
Iran, Islamic Rep. of	5 (0.8)	5 (0.7)	7 (0.9)	74 (1.8)	10 (1.1)
Italy	7 (1.0)	11 (1.5)	8 (1.1)	65 (2.2)	9 (1.4)
Slovenia	10 (1.2)	11 (1.3)	13 (1.0)	63 (2.0)	4 (0.8)
Armenia	8 (1.5)	9 (1.4)	13 (1.8)	60 (2.5)	10 (1.3)
Norway	10 (0.8)	14 (1.4)	18 (1.2)	49 (1.8)	9 (0.9)
Philippines	19 (1.1)	17 (1.2)	17 (1.1)	47 (1.8)	1 (0.2)
Sweden	11 (1.0)	15 (1.3)	25 (1.1)	41 (1.2)	8 (1.0)

* No Response

† Met guidelines for sample participation rates only after replacement schools were included (see Appendix A).

( ) Standard errors appear in parentheses. Because results are rounded to the nearest whole number, some totals may appear inconsistent.

the hypotenuse, *QR*, is the center of the circumscribed circle and that, since *PT* and *QT* are radii of that circle, they must be of equal length. The percent of students choosing the correct response to this item was at least 60 in 7 of the 10 participating countries, and in no country was the percent correct less than 40. The best results were in Lebanon (90%) and the Russian Federation (87%), and approximately three-fourths of the Dutch and Iranian students answered correctly. Non-response rates were quite low, and incorrect responses were distributed across the three alternatives.